Multidimensional Inhomogeneous Cosmology in Scalar Tensor Theory

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Abstract

Exact cosmological solutions are obtained for a five dimensional inhomogeneous fluid distribution along with a Brans-Dicke type of scalar field. The set includes varied forms of matter field including $\rho + p = 0$, where p is the 3D isotropic pressure. Depending on the signature of 4-space curvature our solutions admit of indefinite expansion in the usual 3-space and dimensional reduction of the fifth dimension. Due to the presence of the scalar field the case $p = -\rho$ does not yield an exponential expansion of the scale factor, which strikingly differs from our earlier investigations without scalar field.The *effective* four dimensional values of entropy and matter are calculated and possible consequences of entropy and matter generation in the 4D world as a result of dimensional reduction of the extra space are also discussed. Encouraging to point out that aside from the well known big bang singularity our inhomogeneous cosmology is spatially regular everywhere. Further our model seems to suggest an alternative mechanism pointing to a smooth pass over from a primordial, inhomogeneous cosmological phase to a 4D homogeneous one.

KEYWORDS : cosmology; higher dimensions; scalar field

1. Introduction

There has been of late a resurgence of interests in theories both in general relativity and particle physics where the dimension of space time is greater than the usual (3+1) we observe. These theories include kaluza-klein , induced matter, super string, supergravity and string. The interest in higher dimensional spacetime also stems from its recent applications to brane cosmology (see ref. 1 for a lucid exposition of these ideas) [1]. In these (4+d) dimensional models the d-spacelike dimensions are generally spontaneously compactified and the symmetries of this space appear as gauge symmetries of the effective 4D theory.

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Homogeneous Kaluza-Klein extension of the FRW model has been fairly adequately discussed in the literature [2]. Starting from a topology of $R^1 \times R^3 \times S^d$ it is shown that both the *standard* and the extra space expand initially after which the curvature effects become significant and the extra compact space collapses to a singularity. It is conjectured that some sort of quantum gravity effect produces a repulsive potential to stabilize the compact space at the planckian length and there after the visible universe expands in the FRW way.

However, inhomogeneous cosmological models in higher dimensions have not so far attracted the attention it deserves. But the analysis made by de Lapparent et al [3] of the *cfa* redshift survey and also the observations of Saunders et al [4] of Infrared Astronomy Satellite survey indicate that the large scale structure of the universe does not show itself as a smooth and homogeneous distribution of matter as was believed earlier. At the same time the failure of the theoretical considerations such as statistical fluctuations in the FRW models to explain the large scale structure suggests that the inhomogeneity factor in any physical cosmological model can no longer be avoided. Motivated by these considerations some of us have attempted to study inhomogeneous models and their implications in a series of papers in 5D space time [5].

On the other hand, it is known that the idea of inflation tries to address the twin problems of horizon and flatness that plague the standard big bang model. It is conjectured that the vacuum energy driving the inflation is associated with a scalar field called inflaton. The futile effort to identify the inflaton in the particle sector and the fact that the gravitational force is the only long range force governing the evolution of the universe leads us to consider inflation as a pure gravitational effect. This as well as the problem associated with the 'graceful exit' criteria led people to seriously reconsider the Brans-Dicke theory. The recent interest in B-D theory also stems from its relation to low energy bosonic string theory. It is observed that the two theories give identical predictions when the B-D coupling constant $\omega = -1$. All these considerations have led to several recent investigations into the generation of exact solutions for Cosmology [6].

The present work is primarily motivated by our desire to investigate the influence of B-D field in the frame work of higher dimensional inhomogeneous space time assuming varied equations of state. The paper is organized as follows: In section 2 the field equations and their integrals are found where we observe that as the 3D scale factor expands the extra dimension shrinks with time. This is desirable in the context of any higher dimensional theory. In this section we also discuss the specific cosmological evolution assuming different equations of state. The case $p = -\rho$ is strikingly different in the sense that here we do not get exponential expansion of the scale factor as is customary under identical situations. This may be due to the presence of the scalar field. Here we have also studied the specifics of our model as regards the effective entropy generation, matter creation etc in the 4D world. The paper ends with a discussion in section 3.

2. The Field Equations and Their Integrals

The 5D line-element is taken as

$$ds^2 = dt^2 - R^2(dr^2 + r^2 d\theta^2 + r^2 sin^2\theta d\phi^2) - B^2 dy^2 \qquad (1)$$

where $R = R(t)$ and $B = B(r,t)$ and y is the fifth co-ordinate. So here we assume that the physical 3-space is both flat and homogeneous while inhomogeneity is being introduced through the extra space. This, however, makes the total 5D space time an inhomogeneous one. One should also note that since $B = B(r,t)$ we are not dealing here with a product space, the shape of the extra space is different at different points of the 4D world. We shall subsequently see that the fact that inhomogeneity is being introduced via the extra space has far reaching implications in the cosmological evolution of our models.

We shall write down the field equations in Dicke's revised units (Einstein frame)[7] as

$$G_{ij} = R_{ij} - \frac{1}{2}g_{ij}R = -(T_{ij} + \Lambda_{ij}) \tag{2}$$

$$\Lambda_{ij} = \frac{2\omega+3}{2\psi^2}(\psi_i\psi_j - \frac{1}{2}g_{ij}\psi_k\psi^k) \tag{3}$$

where $\psi_{,i} \equiv \psi_i$

$$\Box(\log\psi) = \frac{T}{2\omega+3} \tag{4}$$

Here T_{ij} is the usual energy momentum tensor for the matter field and Λ_{ij} is that due to the scalar field in revised units. In what follows we assume for simplicity that the scalar field depends on time only, i.e., $\psi = \psi(t)$. The only non vanishing components of Λ_{ij} are

$$-\Lambda^0_0 = \Lambda^1_1 = \Lambda^2_2 = \Lambda^3_3 = \Lambda^4_4 = -\frac{2\omega+3}{4}(\frac{\dot\psi}{\psi})^2 = -\frac{k}{2}(\frac{\dot\psi}{\psi})^2 \tag{5}$$

where $k = \frac{2\omega+3}{2}$. In comoving co-ordinates the field equations (2) for the metric(1) are given by

$$\begin{aligned}
G^1_0 &= \frac{\dot B'}{B} - \frac{B'\dot B}{B^2} - \frac{\dot R B'}{RB} = 0 && (6)\\
G^1_1 &= \frac{2}{R^2 r}\frac{B'}{B} - (2\frac{\ddot R}{R} + \frac{\dot R^2}{R^2} + \frac{\dot B^2}{B^2} + 2\frac{\dot R\dot B}{RB}) = p + \frac{k}{2}(\frac{\dot\psi}{\psi})^2 && (7)\\
G^2_2 &= G^3_3 = \frac{1}{R^2}(\frac{B''}{B} + \frac{1}{r}\frac{B'}{B}) - (2\frac{\ddot R}{R} + \frac{\dot R^2}{R^2} + \frac{\dot B^2}{B^2} + 2\frac{\dot R\dot B}{RB}) = p + \frac{k}{2}(\frac{\dot\psi}{\psi})^2 && (8)\\
G^4_4 &= -3(\frac{\ddot R}{R} + \frac{\dot R^2}{R^2}) = p_5 + \frac{k}{2}(\frac{\dot\psi}{\psi})^2 && (9)\\
G^0_0 &= \frac{1}{R^2}(\frac{B''}{B} + \frac{2}{r}\frac{B'}{B}) - 3(\frac{\dot R^2}{R^2} + \frac{\dot R\dot B}{RB}) = -\rho - \frac{k}{2}(\frac{\dot\psi}{\psi})^2 && (10)
\end{aligned}$$

and the wave equation is

$$\frac{\ddot\psi}{\psi} - \frac{\dot\psi^2}{\psi^2} + 3\frac{\dot\psi}{\psi}\frac{\dot R}{R} + \frac{\dot\psi}{\psi}\frac{\dot B}{B} = \frac{T}{2k} \tag{11}$$

(where T is the trace of energy-momentum tensor). From equation(6)

$$B = \beta(r)R + \alpha(t) \tag{12}$$

where $\beta(r)$ and $\alpha(t)$ are integration function of r and t respectively, and from equation (7) and (8)

$$B = -c\gamma(t)r^2 + \alpha(t) \tag{13}$$

where c is a constant of integration and $\gamma(t)$ is integration function of t. Now comparing equation (12) and (13) we finally get

$$B = -cr^2 R + \alpha(t) \tag{14}$$

CASE I ($p = p_5 = 0$: dust distribution)

It is very difficult to solve the set of equations in a general form. So in view of the fact that the power law expansion in BD theory is now being seriously considered as a possible scenario to solve the 'graceful exit' problem in inflationary cosmology we assume $R \sim t^a$, where 'a' is a constant. Now eliminating ρ from equations(10) and (11) we get via equation (14) the following two relations

$$3\frac{\dot{R}^2}{R^2} = k(\frac{\ddot{\psi}}{\psi} - \frac{3}{4}\frac{\dot{\psi}^2}{\psi^2}) + 4k\frac{\dot{R}\dot{\psi}}{R\psi} \tag{15}$$

$$(3\frac{\dot{R}}{R} - 2k\frac{\dot{\psi}}{\psi})\dot{\alpha} + \{3\frac{\dot{R}^2}{R^2} - 2k(\frac{\ddot{\psi}}{\psi} - \frac{3}{4}\frac{\dot{\psi}^2}{\psi^2} + 3\frac{\dot{R}\dot{\psi}}{R\psi})\}\alpha + 6\frac{c}{R} = 0 \tag{16}$$

From equation (9) we also get

$$\psi = t^{\sqrt{\frac{6a(1-2a)}{k}}} \tag{17}$$

The equation (15) further fixes the value of 'a' to be either $a = \frac{8k}{3+16k}$ or $a = \frac{1}{4}$. But in order to be consistent with other equations the first value is to be discarded and also $k = \frac{3}{4}$ (i.e., $\omega = -\frac{3}{4}$). For economy of space we skip all intermediate steps and solve equation (16) to find α as

$$\alpha(t) = bt^{\frac{1}{4}} + \frac{16c}{4\sqrt{3k} - 3}t^{\frac{7}{4}} \tag{18}$$

So all pieces put together finally yield

$$R = t^{\frac{1}{4}} \tag{19}$$

$$B = (b - cr^2)t^{\frac{1}{4}} - \frac{16c}{9}t^{\frac{7}{4}} \tag{20}$$

$$\psi = \frac{1}{t} \tag{21}$$

$$\rho = \frac{36c}{9(b - cr^{2)t^{\frac{1}{2}}} - 16ct^2} \tag{22}$$

From the equation (22) it follows that $\rho > 0$ implies that $c > 0$. One can, at this stage, calculate the 4-space curvature of the t-constant hyper surface for the line element (1). The relevant expression is given by [8]

$$R_i^i = R^{*(4)} + \dot{\theta} + \theta^2 - 2\omega^2 + \dot{u}^i_{;i} \tag{23}$$

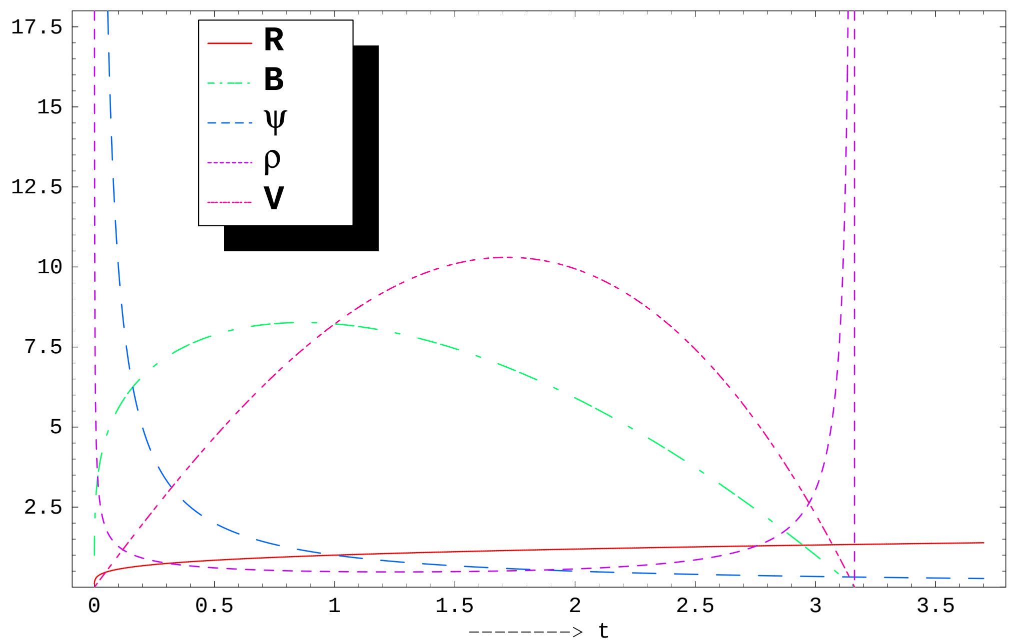

Figure 1: *Plot of R, B, ψ, ρ and V vs time. The value of the parameter c has been taken to be 1, and $b = r^2 + 10$.*

where θ is the expansion scalar and the last two terms refer to vorticity and acceleration and $R^{*(4)}$ is the 4-space curvature.

After some algebra we get

$$R^{*(4)} = \frac{12c}{(b - cr^2)t^{\frac{1}{4}} - \frac{16c}{9}t^{\frac{7}{4}}} \tag{24}$$

Thus we can fix up the arbitrary constant c as a measure of the curvature of the 4-space. As is evident from the expression (24) this curvature, unlike the homogeneous FRW model is a function of the spatial co-ordinates also and it blows off at the initial singularity t=0. Otherwise it is regular everywhere. The shear scalar also comes out to be

$$\sigma^2 = \sigma_{ij}\sigma^{ij} = \frac{3}{2}\left(\frac{\dot{B}^2}{B^2} - \frac{\dot{R}^2}{R^2}\right) = \frac{8c^2t^{\frac{3}{2}}}{3B^2} \tag{25}$$

So with time the model increasingly anisotropises as the denominator reduces to planckian length. This is only to be expected in any sensible multidimensional model.

To check if our space time contains any geometric singularity aside from the well known big bang one at t=0, we calculate the Kretschmann scalar as $R_{ijkl}R^{ijkl}$. Explicit calculations (also checked and verified with the help of a computer-aided program) show that only the R_{0441} survives such that the scalar reduces to

$$R_{ijkl}R^{ijkl} = \frac{2}{R^2B^2}(B^{''})^2 \tag{26}$$

Nice thing about it is that for our model the scalar is regular everywhere including the point r=0, which may be identified with the centre of the fluid distribution as in many other inhomogeneous distributions.

Dynamical Behaviour:

The behaviour is schematically shown in the adjoining figure (1). The spatial volume is given as

$$V = R^3B = (b - cr^2)t - \frac{16c}{9}t^{\frac{5}{2}} \tag{27}$$

For positive curvature ($c > 0$) the extra space collapses at a finite time $t = t_c$. If the extra dimension does collapse to zero in a finite time we can not analytically continue time beyond that in the usual case. A possible way out of the difficulty is to find out some sort of stabilizing mechanism which might fix up the extra scale at a planckian length.

Moreover, if we consider the time evolution of a shell characterized by a r-const. hyper surface it follows that for $c > 0$, the 4-volume (R^3B) collapses. This also follows from equation (24) as $R^{*(4)} > 0$ for $c > 0$. Interestingly both the spatial volume and the extra dimension collapse at the same instant (fig 1).

One difference from the analogous homogeneous case is too obvious. While in the homogeneous case the whole distribution collapses at a single instant, here the collapse time depends on the radial co-ordinate. So the big crunch is not simultaneous for all the shells.

Entropy Generation:

Before concluding the section let us attempt to address a particular problem of current interest - the very high value of entropy per baryon observable at present. If one assumes that the 4D formulation of the laws of thermodynamics can be extended to 5D also and the interactions at the early universe led to thermodynamical equilibrium, the total entropy of the universe in an adiabatic change can not but conserve. Now S_4, the effective four dimensional entropy that we observe at present is given by

$$T\frac{dS_4}{dt} = \frac{d}{dt}(\rho R^3) + p\frac{d}{dt}(R^3) \tag{28}$$

Again divergence of equation (2) gives

$$\dot{\rho} + (\rho + p)\frac{3\dot{R}}{R} + (\rho + p_5)\frac{\dot{B}}{B} + \frac{1}{2}(\rho - 3p - p_5)\dot{\psi} = 0 \tag{29}$$

such that

$$T\dot{S}_4 = -(\rho + p_5)\frac{\dot{B}}{B} - \frac{1}{2}(\rho - 3p - p_5)\dot{\psi} \tag{30}$$

In our model both $\dot{B}$ and $\dot{\psi}$ are negative giving $\dot{S}_4 > 0$ for physically reasonable matter field. So the observable entropy increases as the extra dimension shrinks.And the effect is augmented due to the presence of the scalar field. The above situation is discussed for homogeneous distribution without scalar field by Alvarez and Gavela [9] in an earlier work in the context of a homogeneous model.

Matter Leakage:

One may look at the conservation relation (29) from another stand point also. We know that to avoid big bang singularity Gold and Bondi [10] hypothesized continuous creation of matter without offering any dynamical mechanism for it. Later Hoyle and Narlikar [11] invoked an extraneous 'creation field' in their C-field theory for matter creation. In both the cases the conservation principle is clearly given up. It may be tempting to suggest an alternative scenario where conservation principle strictly holds in the 5D world. For our dust case and in absence of scalar field the equation (29) leads to

$$\rho R^3 B = Constant = M(0) \tag{31}$$

where M(0) may be identified with the total mass in the 5D world which strictly conserves. But for a 4D observer the effective 4D matter is given by $\rho R^3 = M_4$, such that

$$M_4(t) = M_0 B^{-1} \tag{32}$$

Since for a realistic model the extra dimension shrinks the above equation tells us that although the overall (4+1)D matter remains conserved there will be matter leakage from the internal space onto the effective 4D world. So it offers a natural mechanism of matter creation in 4D space time without the assumption of an extraneous field [12]as also without violating any conservation principle in physics.

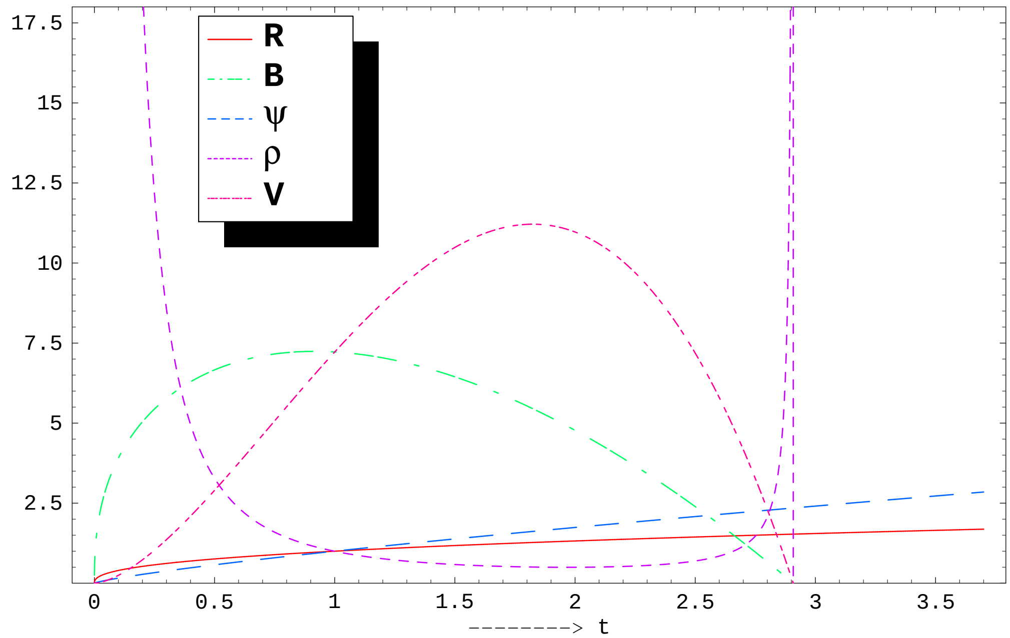

Figure 2: *Plot of R, B, ψ, ρ and V vs time. The value of the parameter c has been taken to be 1, $k = 3/4$ and $b = r^2 + 10$.*

Case II : $p \neq p_5 \neq 0$

While working with matter field in higher dimensional theories three possibilities present themselves : (i) a vacuum state, (ii) a low temperature state, i.e. , $T < \frac{1}{B}$ where evidently $\frac{1}{B}$ gives the curvature scale of S^1, and (iii) a high temperature state, i.e. , $T > \frac{1}{B}$. In this case we take the third possibility where the energy of the particles is higher than the excitation energy of the internal space such that the higher excitations in the internal space can no longer be neglected. Therefore, the higher dimensional stress $p_5 \neq 0$ and the isometry of our metric dictates that in the comoving co-ordinates $T^1_1 = T^2_2 = T^3_3 = -p$. In this case the wave equation is

$$\frac{\ddot{\psi}}{\psi} - \frac{\dot{\psi}^2}{\psi^2} + (3\frac{\dot{R}}{R} + \frac{\dot{B}}{B})\frac{\dot{\psi}}{\psi} = \frac{\rho - 3p - p_5}{2k} \tag{33}$$

For simplicity we assume $\frac{\dot{\psi}}{\psi} = 2\frac{\dot{R}}{R}$ and for economy of space we skip all intermediate mathematical steps to finally get

$$R = t^{\frac{2}{5}} \tag{34}$$

$$B = (b - cr^2)t^{\frac{2}{5}} - \frac{75c}{33 - 8k}t^{\frac{8}{5}} \tag{35}$$

$$
\begin{aligned}
\psi &= t^{\frac{4}{5}} && (36)\\
p &= \frac{(\frac{6-8k}{25})(b-cr^2)t^{-\frac{8}{5}} + \frac{(12+56k)}{33-8k}ct^{-\frac{2}{5}}}{(b-cr^2)t^{\frac{2}{5}} - \frac{75c}{(33-8k)}t^{\frac{8}{5}}} && (37)\\
\rho &= \frac{(\frac{24-8k}{25})(b-cr^2)t^{-\frac{8}{5}} + \frac{(18-24k)}{33-8k}ct^{-\frac{2}{5}}}{(b-cr^2)t^{\frac{2}{5}} - \frac{75c}{(33-8k)}t^{\frac{8}{5}}} && (38)\\
& && (39)\\
p_5 &= \frac{6-8k}{25}t^{-2} && (40)
\end{aligned}
$$

the only restriction being $(2\omega+3) < \frac{3}{2}$.

The temporal behaviour in this model is shown in figure (2). Here the 3D space expands indefinitely and the extra dimension is amenable to reduction. Further if the 4D curvature $c = 0$ we get $R = B$ and the model becomes isotropic and homogeneous and interestingly $p = p_5$. Further to mention that when $k = \frac{3}{4}$ the five dimensional pressure vanishes.

Case III : $p = -\rho$**,** $p_5 = 0$

From equation (7) and (10),

$$
\begin{aligned}
\frac{\ddot{R}}{R} - \frac{\dot{R}^2}{R^2} &= -\frac{k}{3}(\frac{\dot{\psi}}{\psi})^2 && (41)\\
\ddot{\alpha} - \frac{\dot{R}}{R}\dot{\alpha} + \{2(\frac{\ddot{R}}{R} - \frac{\dot{R}^2}{R^2}) + k(\frac{\dot{\psi}}{\psi})^2\}\alpha &= \frac{2c}{R} && (42)
\end{aligned}
$$

Again as $p_5 = 0$, we get from equations (41) and (9)

$$
\begin{aligned}
R &= t^{\frac{1}{4}} && (43)\\
\psi &= t^{\sqrt{\frac{3}{4k}}} && (44)
\end{aligned}
$$

while equation (42) yields

$$\alpha = bt^{\frac{1}{4}} + \frac{16c}{9}t^{\frac{7}{4}} \qquad (45)$$

Here also we have a restriction on k as $k = 48$, i.e. , $\omega = \frac{93}{2}$ and

$$
\begin{aligned}
B &= (b-cr^2)t^{\frac{1}{4}} + \frac{16c}{9}t^{\frac{7}{4}} && (46)\\
p &= -\rho = -\frac{72c}{9(b-cr^2)t^{\frac{1}{2}} + 16ct^2} && (47)
\end{aligned}
$$

The equation (47) further dictates that as ρ and B are non negative the constant c should be greater that zero.Hence dimensional reduction in this model is clearly ruled out.There are two striking features in this case. Firstly the $'\rho'$ and $'p'$ are not separately constant as in 4D case. This, however, follows from the divergence relation (29). Secondly

there is no exponential expansion of the 3D scale factor. In our earlier work [5] we have got exponential expansion of the 3D scale factor in 5D model without any scalar field. So the presence of the scalar field makes the difference.

3. Discussion :

Before concluding let us sum up the basic features of our model. Here we consider an inhomogeneous 5D cosmology where the inhomogeneity is tacitly introduced through the extra fifth dimension. Encouraging to point out that apart from the well known big bang singularity we do not encounter any spatial singularity including the centre of symmetry (r=0) as evidenced from the regularity of the kreschmann scalar. Secondly the extra dimension exhibits dimensional reduction, a property which is always sought for while working with multidimensional cosmology.

Moreover,the recent COBE DMR experiments [13] suggest a scale of temperature fluctuation, which may be related to density fluctuation, thus providing unambiguous evidence for the existence of inhomogeneity in the early universe. Since the visible world around us is manifestly homogeneous and isotropic, various models have been proposed to account for a possible pass over from the primordial inhomogeneous scale to the current homogeneous one. Explicit computations [14] however, showed that the so called *'chaotic'* models [15] can not explain these observed cosmological properties. The scenario presented in this work seem to provide an alternative resolution to this problem as follows : It is believed that the extra dimension once contracting will ultimately stabilize at a very small length and after that the extra dimension will lose its dynamical character. But an unphysical nature of this model is that it is not apparent from our analysis how that stabilization works. However, if the number of extra dimensions is at least two it may generate a repulsive potential to halt the contraction at a certain stage [16].

Whatever be the stabilizing mechanism if one believes that it does occur then for our model it has far reaching consequences. Because as the fifth dimension becomes constant then our model becomes exactly FRW type. So not only do we enter a 4D world it also envisages a smooth transition from a 5D inhomogeneous cosmology to 4D homogeneous one. And this homogenization takes place without forcing us to choose very special initial conditions as is customary in other standard 4D models.

To end a final remark may be in order.In some of the cases discussed above the B-D coupling constant ω comes out to be negative. This is not as undesirable aspect as was thought to be earlier. Because as pointed out in the introduction the low energy sector of the string theory contains an action which may be identified with the B-D action subject to the restriction that $\omega = -1$. Further the much discussed quintessence phenomenon in the frame- work of scalar tensor theory suggests that for the accelerated expansion of the Universe the ω should be negative even when an additional potential field is introduced in the action.

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